Correlations within eigenvectors and transition amplitudes in the two-body random interaction model

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It is shown that the two-body character of the interaction in a many-body system gives rise to specific correlations between the components of compound states, even if this interaction is completely random. Surprisingly, these correlations increase with the increase of the number of active (valence) particles. Statistical theory of transition amplitudes between compound states, which takes into account these correlations is developed and tested within the framework of the two-body random interaction model. It is demonstrated that a feature, which can be called ''correlation resonance,'' appears in the distribution of the transition matrix amplitudes, since the correlations strongly reduce the transition amplitudes at the tails and increase them near the maximum of the distribution.

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I. INTRODUCTION

Measurements of parity nonconservation in neutron capture by Th nucleus [1] gave a surprising result: in spite of a natural assumption of a random character of matrix elements of the weak interaction between compound states, the effect was found to be of the same sign for all observed resonances. Possibly, this means that the strongly fluctuating matrix elements of a weak perturbation between ''chaotic'' states of a compound nucleus (which was, in fact, the first example of a quantum chaotic system) are essentially correlated. Thus the problem of correlations between components of compound states is of great importance both for theory and applications.

An attempt to study these correlations has been undertaken in [2–4]. It is natural to expect that some correlations may appear if the number of independent parameters in a Hamiltonian matrix is substantially smaller than the total number of the Hamiltonian matrix elements. The simplest example is given by the model of a random separable interaction [5] (see also [4]),

$$H_{ij}=\epsilon_i\delta_{ij}+gv_iv_j, \quad i,j=1,2,\ldots,N \tag{1}$$

where ϵ_i are the unperturbed energies and v_i are random variables distributed, e.g., according to a Gaussian law. As one can see, the number of independent parameters v_i is equal to N, while the number of the Hamiltonian matrix elements H_{ik} is N^2. It was shown that this model displays very strong ($\sim 100\%$) correlations between eigenvectors with close energies, despite the random character of the interaction: $\langle v_iv_j\rangle=\delta_{ij}\langle v_i^2\rangle$.

Such correlations cannot appear in models described by full random matrices, like those of the Gaussian orthogonal ensemble (GOE). It was always obvious that such models possess some unphysical features, e.g., the semicircle level density, however, they seem to give a very accurate description of the correlations and fluctuations of energy levels (see, e.g. [6]). The important point is that in real many-body systems the basic interaction is a two-body one. This means that the number of independent parameters determining the n-particle Hamiltonian (two-body matrix elements) is much smaller than the number of the Hamiltonian matrix elements. Taking the two-body matrix elements as Gaussian random variables, a model called the two-body random ensemble (TBRE) was introduced in [7,8] (see also references in [6]). This model looks in principle much more realistic than the GOE. In particular, it has a Gaussian form of the level density, which is in good agreement with various nuclear shell-model calculations for ''realistic'' interactions in the finite basis (see, e.g., [9]). The TBRE does not allow a deep analytical treatment, however, numerical modeling showed that its level fluctuation properties are very close to those of the GOE, although some differences were noticed [8]. Therefore, in some respect, the two-body nature of the particle interaction does not reveal itself in the level statistics. In other words, level fluctuations are insensitive to the details of the interaction between particles, provided the latter is large enough to cause strong mixing of the basis states.

Another model which seems to be more physical than the GOE was proposed a while ago by Wigner [10] to describe compound nuclei. It was suggested that the Hamiltonian matrix H_{ij} has a banded structure, i.e., all matrix elements with $|i-j|>b$ are zeros (b is the bandwidth). The matrix elements inside the band are random numbers with zero mean and fixed variance, $\overline{H_{ij}}=0$, $\overline{H_{ij}^2}=V^2$, except those on the main diagonal, which monotonically increase, $H_{ii}=iD$. In this model all eigenstates are localized in the unperturbed basis ($V=0$). In the nonperturbative regime, $1<V/D<\sqrt{b}$, the strength function which describes the localization in the energy space (also called the local spectral density of states), has a characteristic Breit-Wigner form with a width $\Gamma=2\pi V^2/D$ within the band. This shape is in agreement with nuclear data [11], and with the calculated localization properties of chaotic eigenstates in the rare-earth atom of Ce

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[12]. It has also been shown in [12,13] that the Hamiltonian matrix which produces the dense spectrum of atomic excited states in Ce is sparse and has a bandlike structure, although the edges of the band are diffuse. We should also mention that a number of analytical results on the localization properties of such band random matrices (BRM) have recently been obtained in [14,15] (see also [16], and references therein). However, the off-diagonal Hamiltonian matrix elements in this BRM model are independent random variables, thus this model is void of any possible correlations related to the two-body interaction between particles.

In this paper we show that there are quantities (transition amplitudes or transition strengths) for which the underlying two-body interaction is of crucial importance. We show that such an interaction gives rise to specific correlations between the components of eigenstates, which are very essential for the distribution of transition strengths. Our results are obtained in the framework of the two-body random interaction model (TBRIM) recently proposed in [17] for the study of various physical problems related to such complex many-body systems as heavy atoms, nuclei, metallic clusters, etc., which display quantum chaotic behavior. Being in some aspects similar to the TBRE, the TBRIM is simpler in the sense that it abandons all restrictions imposed by the conservation of the angular momentum, which makes it closer to the embedded GOE [18]. On the other hand, the nondegenerate spectrum of the single-particle orbitals the TBRIM is based upon generates a realistic level density and leads to a bandlike structure of the Hamiltonian matrix.

We should mention that there were quite a number of earlier works where strength distributions were studied using statistical spectroscopy methods and nuclear shell-model calculations ([19], see also review [6]). These methods are based on the calculation of distribution moments, which are given by traces of products of the operators in question and powers of the Hamiltonian over the model finite-dimensional space of the problem. Since the calculation of traces does not require knowledge of eigenstates, the question of correlations within eigenstates which is of prime importance for the present work has not been addressed in those studies. We must add that statistical spectroscopy methods emphasize and employ a particular importance of Gaussian spreading of many-particle configurations, and features like Breit-Wigner localization either do not appear, or are neglected (together with the interaction between configurations) in that formalism. All in all, it is unfortunately very difficult for the present approach to make contact with those results. Comparing the two approaches we should say that at first sight ours does not look as rigorous and mathematically advanced as the other one, as it appeals to some heuristic arguments and uses rather simple mathematics, e.g., perturbation theory. However, we believe that, supported by numerical experiments, our method can give a deeper insight and a more physical picture of transitions between and correlations within the chaotic eigenstates in complex many-body systems.

In Sec. II of this paper we show how the basic two-body interaction results in the correlations between the Hamiltonian matrix elements, eigenstate components, and transition amplitudes. In Sec. III we check whether the effects found in Sec. II could lead to some correlations between transition amplitudes coupling different pairs of states. Section IV presents a brief outline of a statistical approach to the calculation of transition strengths; the analytical results are checked there against numerical ones obtained in the TBRIM. Finally, in Sec. V we study the spreading widths of the many-particle basis states.

II. CORRELATIONS BETWEEN EIGENVECTOR COMPONENTS INDUCED BY TWO-BODY INTERACTION

Let us consider the basic ideas of the TBRIM. In this model, n Fermi particles are distributed among m nondegenerate orbitals. In doing numerical experiments, we assume, as in [17], that the energies of the orbitals are given by the simple expression

$$\epsilon_\alpha = d_0\left(\alpha+\frac{1}{\alpha}\right), \quad \alpha=1,2,\ldots,m. \tag{2}$$

However, the analytical treatment presented below does not depend on a particular form of ϵ_α. Many-particle basis states $|i\rangle$ are constructed by specifying the n occupied orbitals. The energy E_i of the basis state equals the sum of the single-particle energies over the occupied orbitals. The total number of the many-particle states in the model is $N=m!/n!(m-n)!\sim\exp[n\ln(m/n)+(m-n)\ln(m/m-n)]$. The latter estimate relates to large m and n and shows that N is exponentially large for $n,m-n\gg 1$.

The number of independent parameters of the many-body Hamiltonian is given by the number of different two-body interaction matrix elements $V_{\alpha\beta\gamma\delta}$ and equals $N_2=m^2(m-1)^2/2$. Due to the two-body character of the interaction, the Hamiltonian matrix element $H_{ij}=\langle i|H|j\rangle$ is nonzero only when $|i\rangle$ and $|j\rangle$ differ by no more than two occupied single-particle orbitals. As a result, the number K of the nonzero matrix elements H_{ij} is given by

$$K=N(K_0+K_1+K_2),$$

$$K_0=1, \quad K_1=n(m-n),$$

$$K_2=\frac{1}{4}n(n-1)(m-n)(m-n-1), \tag{3}$$

where K_0, K_1, and K_2 are the numbers of the Hamiltonian matrix elements coupling a particular basis state i to another one, j, which differs from i by the positions of none, one, and two particles, respectively. Therefore for $n,m-n\gg 1$ we have $N_2\ll K\ll N^2$, i.e., the Hamiltonian matrix is essentially sparse and, in a sense, strongly correlated.

To see the correlation between nonzero matrix elements, let us consider a pair of basis states $|i\rangle$ and $|j\rangle$ which differ by the states of two particles, for example, the state $|j\rangle$ can be obtained from the state $|i\rangle$ by transferring the particles from the orbitals α,β into the orbitals γ,δ. For all such pairs, the Hamiltonian matrix elements are the same, $H_{ij}=V_{\alpha\beta\gamma\delta}$ (or, strictly speaking, $H_{ij}=\pm V_{\alpha\beta\gamma\delta}$, due to Fermi statistics). It is easy to calculate the total number N_{eq} of the matrix elements H_{ij} equal to $V_{\alpha\beta\gamma\delta}$, using the fact that the remaining $n-2$ particles can be arbitrarily distributed over $m-4$ orbitals,

$$N_{eq}=\frac{(m-4)!}{(n-2)!(m-n-2)!}. \quad (4)$$

For basis states $|i\rangle$ and $|j\rangle$ which differ by the state of one particle ($\alpha\rightarrow\beta$) the matrix element H_{ij} equals the sum of the $n-1$ two-body interaction matrix elements, $H_{ij}=\Sigma_\gamma V_{\alpha\gamma\beta\gamma}$ (the index γ runs over the rest $n-1$ occupied orbitals). In this case H_{ij} for different $|i\rangle$ and $|j\rangle$ (with fixed α and β) do not coincide, but may contain identical terms $V_{\alpha\gamma\beta\gamma}$, i.e., they are also correlated.

The eigenstates $|n_1\rangle$ of the model are determined by their components $C_i^{(n_1)}$ with respect to the many-particle basis states $|i\rangle$,

$$|n_1\rangle=\sum_i C_i^{(n_1)}|i\rangle, \quad (5)$$

and can be found by solving the Schrödinger equation,

$$\sum_j H_{ij}C_j^{(n_1)}=E^{(n_1)}C_i^{(n_1)}. \quad (6)$$

If the perturbation V is strong enough, the exact eigenstates $|n_1\rangle$ are superpositions of a large number of basis states. As is known, strong mixing of basis states in the exact eigenstates (compound states) occurs locally within some energy range, $|E_i-E^{(n_1)}|\leqslant\Gamma$, where Γ is known as the ''spreading width.'' It can be estimated as $\Gamma\approx N_w D$, where D is the local mean level spacing for many-particle states and N_w is the effective number of basis states represented in a compound state. This number is also known as the number of ''principal components.'' These components give the main contribution to the normalization condition $\Sigma_i|C_i^{(n_1)}|^2=1$ for the eigenstate $|n_1\rangle$. Formally, we can estimate N_w as the reciprocal of the inverse participation ratio, $N_w^{-1}\simeq\Sigma_i|C_i^{(n_1)}|^4$.

It is rather straightforward to show that the correlations between H_{ij} result in correlations between the components $C_i^{(n_1)}$. Indeed, let us multiply the Schrödinger equation by the coefficient $C_i^{(n_1)}$ and sum over n_1. Using the orthogonality condition $\Sigma_{n_1}C_i^{(n_1)}C_j^{(n_1)}=\delta_{ij}$, one obtains

$$H_{ij}=\sum_{n_1} C_i^{(n_1)}E^{(n_1)}C_j^{(n_1)}. \quad (7)$$

In what follows, we assume that the matrix elements of the two-body interaction V are random variables with the zero mean, therefore $\overline{H_{ij}}=0$ for $i\neq j$. In this case one can get $\overline{C_i^{(n_1)}C_k^{(n_1)}}=0$, where the line stands for averaging over different realizations of V. However, if matrix elements of the Hamiltonian are correlated, $\overline{H_{ij}H_{kl}}\neq 0$, the components of different eigenvectors $|n_1\rangle$ and $|n_2\rangle$ are also correlated, since

$$\overline{H_{ij}H_{kl}}=\sum_{n_1n_2} C_i^{(n_1)}E^{(n_1)}C_j^{(n_1)}C_k^{(n_2)}E^{(n_2)}C_l^{(n_2)}\neq 0. \quad (8)$$

The latter relation shows that $\overline{C_i^{(n_1)}C_j^{(n_1)}C_k^{(n_2)}C_l^{(n_2)}}\neq 0$.

The above conclusion has important consequences. Let us consider a single-particle operator

$$\hat{M}=\sum_{\alpha,\beta} a_\alpha^\dagger a_\beta M_{\alpha\beta}=\sum_{\alpha,\beta}\rho_{\alpha\beta}M_{\alpha\beta}, \quad (9)$$

where $a_\alpha^\dagger$ and a_β are the creation and annihilation operators. It is convenient to express the matrix elements of $\hat{M}$ in terms of matrix elements of the density matrix operator $\rho_{\alpha\beta}=a_\alpha^\dagger a_\beta$ which transfers a particle from the orbital β to the orbital α. One can see that the matrix element of $\hat{M}$ between compound states,

$$\langle n_1|\hat{M}|n_2\rangle=\sum_{\alpha,\beta} M_{\alpha\beta}\langle n_1|\rho_{\alpha\beta}|n_2\rangle$$
$$=\sum_{\alpha,\beta} M_{\alpha\beta}\sum_{i,j} C_i^{(n_1)}\langle i|\rho_{\alpha\beta}|j\rangle C_j^{(n_2)} \quad (10)$$

has the zero mean due to the statistical properties of the components, i.e., $\overline{\langle n_1|\rho_{\alpha\beta}|n_2\rangle}=0$. Since the summation over the orbitals α,β in Eq. (10) is independent from the averaging over different realizations of V, in what follows we consider the simplest case of $\hat{M}=\rho_{\alpha\beta}$. The variance of the matrix element of $\rho_{\alpha\beta}$ between the two compound states is equal to

$$\overline{M^2}=\overline{\langle n_1|\rho_{\alpha\beta}|n_2\rangle\langle n_2|\rho_{\beta\alpha}|n_1\rangle}$$
$$=\overline{\sum_{i,j,k,l} C_i^{(n_1)}C_j^{(n_1)}C_k^{(n_2)}C_l^{(n_2)}\langle i|\rho_{\alpha\beta}|k\rangle\langle l|\rho_{\beta\alpha}|j\rangle}$$
$$=S_d^{(n_1n_2)}+S_c^{(n_1n_2)}, \quad (11)$$

where we separated the diagonal and nondiagonal contributions to the sum (11),

$$S_d^{(n_1n_2)}=\overline{\sum_{i,k}|C_i^{(n_1)}|^2|C_k^{(n_2)}|^2|\langle i|\rho_{\alpha\beta}|k\rangle|^2}, \quad (12)$$

$$S_c^{(n_1n_2)}=\overline{\sum_{i\neq j,k\neq l} C_i^{(n_1)}C_j^{(n_1)}C_k^{(n_2)}C_l^{(n_2)}\langle i|\rho_{\alpha\beta}|k\rangle\langle l|\rho_{\beta\alpha}|j\rangle}. \quad (13)$$

Note that the diagonal term $S_d^{(n_1n_2)}$ is essentially positive and can be easily estimated (see [3,12,20] and Sec. IV below), while the nondiagonal term $S_c^{(n_1n_2)}$ is our main interest. If the eigenstates are completely ''random'' (different components both inside each eigenstate and of different eigenstates are uncorrelated), the correlation sum S_c is equal to zero and the variance is determined by the ''diagonal'' sum S_d (this assumption has been used in the previous calculations of matrix elements between compound states in [3,12,20,21]). However, we show below that in a many-body system these two terms are of the same order, $S_c\sim S_d$, even for the random two-body interaction V.

The TBRIM allows one to investigate various properties of chaotic many-body systems taking into account the two-body nature of the interaction between particles. In the previous papers [12,17] there were indications that the diagonal

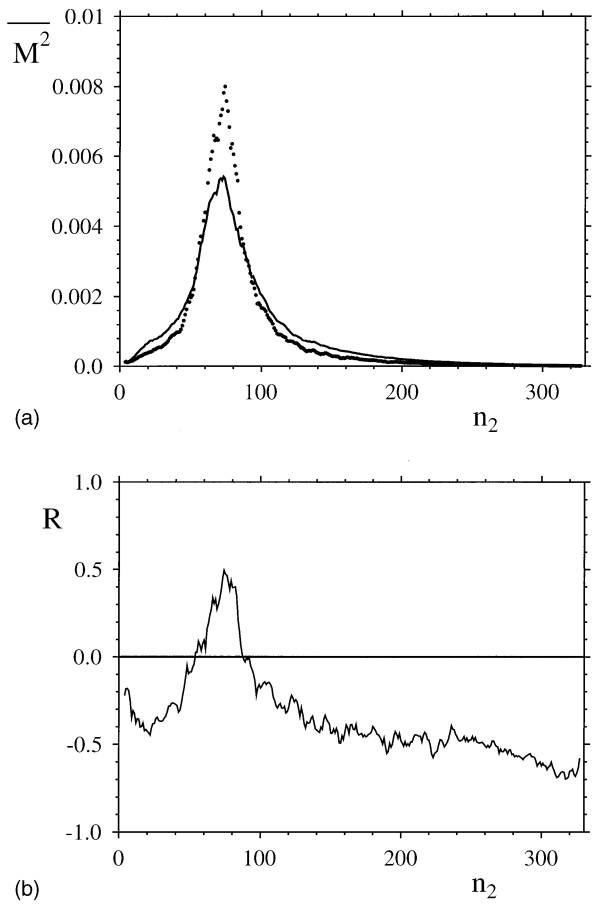

FIG. 1. (a) Mean-square matrix element (11) calculated in the TBRIM for $n=4$ particles and $m=11$ orbitals, $\alpha=4$, $\beta=5$, as a function of the eigenstate n_2 for $n_1=55$. Averaging over $N_r=100$ Hamiltonian matrices H_{ij} for different realizations of the random two-body matrix elements has been made. Dots correspond to the sum S_d+S_c while the solid line represents the diagonal contribution S_d only [see (12)]. (b) Ratio $R=S_c/S_d$ of the correlation contribution to the diagonal contribution.

approximation is not completely accurate for the computation of the variance of matrix elements of perturbation. In order to study this effect in detail, we have performed numerical experiments with TBRIM for the parameters corresponding to the model calculations of the Ce atom [12,13]. We take the number of particles $n=4$, the number of orbitals $m=11$, the spectrum of the single-particle orbitals is determined by $d_0=1$, and the Gaussian random two-body interaction is given by $\sqrt{\overline{V^2}}=0.12$. As a result, the size of the Hamiltonian matrix H_{ij} is $N=330$. The calculation of the matrix elements between compound states in this model gave a remarkable result. In Fig. 1 we present the ''experimental'' value of $\overline{M^2}$ [see Eq. (11)] together with the diagonal contribution (12) [see Fig. 1(a)], and the ratio $R=S_c/S_d$ in Fig. 1(b). Figure 1(a) reveals a systematic difference between the diagonal approximation and exact expression (11), and Fig. 1(b) shows that nondiagonal term S_c is of the same order as S_d, which clearly indicates the presence of correlations.

Below, we show how these correlations emerge in the nondiagonal term S_c. First, note that for a given i the sum over k in Eqs. (12) for S_d contains only one term, for which $|k\rangle=a_\beta^\dagger a_\alpha|i\rangle\equiv|i'\rangle$, determined by transferring one particle from the orbital α to the orbital β in the state $|i\rangle$ (hereafter we will use the notation i' to denote such states). Accordingly, the index i runs over those states in which α is occupied and β is vacant. For such i and i' the matrix element $\langle i|\rho_{\alpha\beta}|i'\rangle=1$, otherwise, it is zero. Therefore, in fact, the sum in (12) is a single sum, with a number of items less than N,

$$S_d^{(n_1n_2)}=\overline{\sum_i{}'\ |C_i^{(n_1)}|^2|C_{i'}^{(n_2)}|^2}, \tag{14}$$

where the sum Σ_i' runs over the specified i. Analogously, Eq. (13) can be written as the double sum over i and j specified as above,

$$S_c^{(n_1n_2)}=\overline{\sum_{i\neq j}{}''\ C_i^{(n_1)}C_j^{(n_1)}C_{i'}^{(n_2)}C_{j'}^{(n_2)}}, \tag{15}$$

where j' is a function of j, $|j'\rangle=a_\beta^\dagger a_\alpha|j\rangle$. Note that the energies of the basis states and their primed partners are connected as $E_{i'}-E_i=\epsilon_\beta-\epsilon_\alpha=E_{j'}-E_j$.

One can expect that maximal values of the sum (14) and, possibly, (15) are achieved when C's are principal components of the eigenstates. This means that the mean square of the matrix element $\overline{|\langle n_1|\rho_{\alpha\beta}|n_2\rangle|^2}$ is maximal when the operator $\rho_{\alpha\beta}$ couples the principal components of the state $|n_1\rangle$ with those of $|n_2\rangle$, i.e., for $E^{(n_1)}-E^{(n_2)}\approx\omega_{\alpha\beta}\equiv\epsilon_\alpha-\epsilon_\beta$. Far from the maximum ($|E^{(n_1)}-E^{(n_2)}-\omega_{\alpha\beta}|>\Gamma$) a principal component of one state, say, n_1, is coupled to a small component k of the other state n_2 ($|E_k-E^{(n_2)}|>\Gamma$). The latter case is simpler to consider analytically, since the admixture of a small component in the eigenstate can be found by means of perturbation theory. This approach reveals the origin of the correlations in the sum S_c, Eq. (15). For example, if $C_j^{(n_1)}$ is a small component of the eigenstate n_1, then it can be expressed as a perturbation theory admixture to the principal components. If $C_i^{(n_1)}$ is one of the latter, then there is a term in the sum (15), which is proportional to the principal component squared, $|C_i^{(n_1)}|^2$.

Indeed, there are three possibilities (i) $C_i^{(n_1)}$ and $C_{j'}^{(n_2)}$ are among the principal components, and $C_j^{(n_1)}$ and $C_{i'}^{(n_2)}$ correspond to the small components. Then, one can write

$$C_j^{(n_1)}=\frac{\langle j|H|\tilde{n}_1\rangle}{E^{(n_1)}-E_j}=\widetilde{\sum_p}\frac{H_{jp}}{E^{(n_1)}-E_j}C_p^{(n_1)}, \tag{16}$$

$$C_{i'}^{(n_2)}=\frac{\langle i'|H|\tilde{n}_2\rangle}{E^{(n_2)}-E_{i'}}=\sum_q\frac{H_{i'q}}{E^{(n_2)}-E_{i'}}C_q^{(n_2)}. \tag{17}$$

The tilde above the sums indicates that the summations run over the principal components only. The ''coherent'' contribution to the sum S_c in Eq. (15) is obtained by separating the squared contributions of the principal components in the sums in $S_c^{(n_1n_2)}$ (i.e., $p=i,q=j'$)

$$\widetilde{\sum_{i,j}}'' \frac{\overline{H_{i'j'}H_{ji}}}{(E^{(n_2)}-E_{i'})(E^{(n_1)}-E_j)}|C_i^{(n_1)}|^2|C_{j'}^{(n_2)}|^2. \quad (18)$$

Taking into account that for the principal components we have $E_i \approx E^{(n_1)}$ and $E_{j'} \approx E^{(n_2)}$, we can replace the energies, $E_{i'} \to E^{(n_1)}+\omega_{\beta\alpha}$, and $E_j \to E^{(n_2)}-\omega_{\beta\alpha}$, and thus obtain the following contribution to $S_c^{(n_1n_2)}$:

$$-\frac{1}{(E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha})^2}\widetilde{\sum_{i,j}}''|C_i^{(1)}|^2|C_{j'}^{(2)}|^2\overline{H_{i'j'}H_{ij}}. \quad (19)$$

(ii) $C_j^{(n_1)}$ and $C_{i'}^{(n_2)}$ correspond to the principal components, $C_i^{(n_1)}$ and $C_{j'}^{(n_2)}$ correspond to the small components. Then, the result is the same as (19).

(iii) $C_i^{(n_1)}$ and $C_j^{(n_1)}$ are principal components, $C_{i'}^{(n_2)}$ and $C_{j'}^{(n_2)}$ are small components (or, $C_{i'}^{(n_2)}$ and $C_{j'}^{(n_2)}$ are principal components, $C_i^{(n_1)}$ and $C_j^{(n_1)}$ are small components). In these cases there are no coherent terms in the sum for S_c in Eq. (13). This follows from the fact that for chaotic eigenstates the mixing among the principal components is practically complete, which makes them to a good accuracy statistically independent.

Thus, far from the maximum, $|E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha}|>\Gamma$, one obtains

$$S_c^{(n_1n_2)} \approx -\frac{2}{(E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha})^2}\widetilde{\sum_{i,j}}''|C_i^{(1)}|^2|C_{j'}^{(2)}|^2\overline{H_{i'j'}H_{ij}}. \quad (20)$$

A similar calculation of the diagonal sum $S_d^{(n_1n_2)}$, Eq. (12), yields

$$S_d^{(n_1n_2)} \approx \frac{1}{(E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha})^2} \times\left[\widetilde{\sum_i}'\widetilde{\sum_{j'}}|C_i^{(n_1)}|^2|C_{j'}^{(n_2)}|^2\overline{H_{i'j'}^2} + \widetilde{\sum_i}\widetilde{\sum_{j'}}'|C_i^{(n_1)}|^2|C_{j'}^{(n_2)}|^2\overline{H_{ij}^2}\right]. \quad (21)$$

The two terms in square brackets result from the contribution of principal i and small i' components in Eq. (14), and vice versa. From Eq. (20) we see that $S_c^{(n_1n_2)}=0$ if $\overline{H_{i'j'}H_{ij}}=0$. However, there is nearly a 100% correlation between these matrix elements. Indeed, the basis state i' differs from i by the location of only one particle (the transition from the orbital α to β), and the same is true for j' and j.

Let us estimate the relative magnitudes of S_d and S_c. First, consider the case when $|i\rangle$ and $|j\rangle$ differ by two orbitals, $|j\rangle=a_{\mu_2}^\dagger a_{\mu_1}a_{\nu_2}^\dagger a_{\nu_1}|i\rangle$. In this case $H_{ij}=V_{\nu_1\mu_1\nu_2\mu_2}$. Since the basis states $|i'\rangle$ and $|j'\rangle$ must differ by the same two orbitals, we have $H_{i'j'}=V_{\nu_1\mu_1\nu_2\mu_2}=H_{ij}$ (note that $\nu_1,\mu_1,\nu_2,\mu_2\neq\alpha,\beta$, since both states $|i\rangle$ and $|j\rangle$ contain α and do not contain β, whereas $|i'\rangle$ and $|j'\rangle$ contain β and do not contain α). Therefore the averages over the nonzero matrix elements between such pairs of states are $\overline{H_{ij}H_{i'j'}}=\overline{H_{ij}^2}=\overline{H_{i'j'}^2}=V^2$.

Now, let us consider the case when $|i\rangle$ and $|j\rangle$ differ by one orbital $|j\rangle=a_{\nu_2}^\dagger a_{\nu_1}|i\rangle$. In this case the Hamiltonian matrix elements are sums of the $n-1$ two-body matrix elements,

$$H_{ij}=\sum_{\mu\neq\alpha}^{n-2}V_{\nu_1\mu\nu_2\mu}+V_{\nu_1\alpha\nu_2\alpha},$$

$$H_{i'j'}=\sum_{\mu\neq\beta}^{n-2}V_{\nu_1\mu\nu_2\mu}+V_{\nu_1\beta\nu_2\beta}.$$

The sums of $n-2$ terms in H_{ij} and $H_{i'j'}$ coincide; the difference is due to the one term only (orbital α is replaced by the orbital β). Thus

$$\overline{H_{ij}H_{i'j'}}=(n-2)V^2,$$

$$\overline{(H_{ij})^2}=\overline{(H_{i'j'})^2}=(n-1)V^2,$$

where we took into account that $\overline{V_{\kappa\lambda\mu\nu}V_{\kappa_1\lambda_1\mu_1\nu_1}}=V^2\delta_{\kappa\kappa_1}\delta_{\lambda\lambda_1}\delta_{\mu\mu_1}\delta_{\nu\nu_1}$.

The contributions of one-particle and two-particle transitions in Eqs. (20) and (21) representing S_c and S_d, respectively, will be determined by the numbers of such transitions allowed by the corresponding sums. For the single-prime sums in Eq. (21) these numbers are proportional to K_1 and K_2, Eq. (3). In the double-prime sum in Eq. (20) these numbers are proportional to $\widetilde{K}_1$ and $\widetilde{K}_2$, the numbers of the two-body and one-body transitions $i\to j$, in the situation when one particle and the two orbitals (α and β) do not participate in the transitions. These numbers can be obtained from Eq. (3) if we replace n by $n-1$, and m by $m-2$, so that $\widetilde{K}_1=(n-1)(m-n-1)$, $\widetilde{K}_2=(n-1)(n-2)(m-n-1)(m-n-2)/4$. Finally we obtain that at $|E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha}|>\Gamma$ the contribution of the correlation term to the variance of the matrix elements of $\rho_{\alpha\beta}$ can be estimated in the ratio as

$$R\equiv\frac{S_c}{S_d}=-\frac{(n-2)\widetilde{K}_1+\widetilde{K}_2}{(n-1)K_1+K_2} =-\frac{(n-2)(m-n-1)(m-n+2)}{n(m-n)(m-n+3)}. \quad (22)$$

This equation shows that for $n=2$ we have $S_c=0$, which is easy to check directly, since $\overline{H_{i'j'}H_{ij}}=0$ in this case. For $n>2$ the correlation contribution S_c is negative at the tails of the strength distribution. This means that it indeed suppresses the transition amplitudes off resonance (see Fig. 1). For $n,m-n\gg1$ the ratio R is approaching its limit value -1. It is easy to obtain from Eq. (22) that for $m-n\gg1$

$$\frac{S_d+S_c}{S_d}=1+R\approx\frac{2m}{n(m-n)}. \quad (23)$$

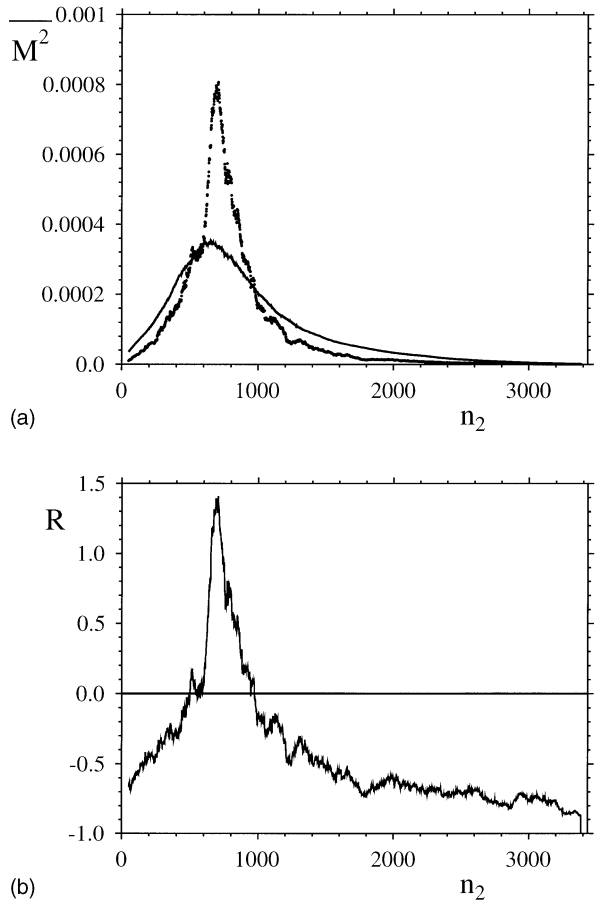

FIG. 2. Same as in Fig. 1, for $n=7$, $m=14$, $\alpha=7$, $\beta=8$. The data obtained for a single Hamiltonian matrix of the size $N=3432$; $n_1=575$. Note the increased role of the correlation contribution S_c.

Thus, surprisingly, the role of the correlation contribution increases with the number of particles.

For the numerical example shown in Fig. 1, $n=4$, $m=11$, one obtains $R=-0.39$, which means that the correlation contribution reduces the magnitude of the squared matrix elements $\overline{M^2}$ between compound states almost by a factor of 2 (for $|E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha}|>\Gamma$). The ratio found numerically is $R\approx-0.45$ [Fig. 1(b), $n_2=150$–250; larger n_2 are probably too close to the boundary of the matrix for R to remain constant].

We would like to stress that the role of the correlation term does not decrease with the increase of the numbers of particles and orbitals. This prediction is supported by Fig. 2, which shows the behavior of the squared matrix element and its diagonal and correlation parts for $n=7$ and $m=14$, $N=3432$. One can see that the suppression of the matrix elements M^2 due to the correlation term at the tails is even stronger than that in Fig. 1 [the numerically found ratio is $R\approx-0.7$ vs $R=-0.55$ obtained from Eq. (22)]. The correlation contribution should be even more important in compound nuclei, where $N\sim10^5$. This case can be modeled by the parameters $n=10$, $m=20$; then we have $R=-0.66$, or, equivalently, $(S_d+S_c)/S_d=0.34$, which means that the correlations suppress the squared element M^2 between compound states by a factor of 3 (far from its maximum).

It is worth emphasizing that the existence of correlations due to the perturbation theory admixtures of small components to the chaotic eigenstates, which leads to a nonzero value of S_c (15), is indeed nontrivial. For example, if one examines the summand of Eq. (15) as a function of i and j, it would be hard to guess that the sum itself is essentially nonzero, since positive and negative values of $C_i^{(n_1)}C_j^{(n_1)}C_{i'}^{(n_2)}C_{j'}^{(n_2)}$ seem to be equally frequent, and have roughly the same magnitude, see Fig. 3.

Since $\Sigma_{n_1}S_c^{(n_1n_2)}=\Sigma_{n_2}S_c^{(n_1n_2)}=0$ (see below), the suppression of $\overline{M^2}$ at the tails should be accompanied by correlational enhancement of the matrix elements near the maximum (at $|E^{(2)}-E^{(1)}-\omega_{\beta\alpha}|<\Gamma$). Thus we come to the important conclusion: even for a random two-body interaction, the correlations produce some sort of a ''correlation resonance'' in the distribution of the squared matrix elements M^2. One should note that this increase of the correlation effects in the matrix elements of a perturbation can be explained by the increased correlations between the Hamiltonian matrix elements when the number of particles and orbitals increases ($N/n\propto e^n$).

Now we can estimate the size of the correlation contribution S_c near the maximum of the the M^2 distribution (at $|E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha}|<\Gamma$). First, we show that after summation over one of the compound states, the correlation contribution vanishes. Indeed,

$$\begin{aligned}\sum_{n_2} S_c^{(n_1n_2)} &= \sum_{n_2}\sum_{i\neq j,k\neq l} C_i^{(n_1)}C_j^{(n_1)}C_k^{(n_2)}C_l^{(n_2)}\langle i|\rho_{\alpha\beta}|k\rangle \\ &\quad\times\langle l|\rho_{\beta\alpha}|j\rangle \\ &= \sum_{i\neq j,k\neq l} C_i^{(n_1)}C_j^{(n_1)}\langle i|\rho_{\alpha\beta}|k\rangle \\ &\quad\times\langle l|\rho_{\beta\alpha}|j\rangle\sum_{n_2} C_k^{(n_2)}C_l^{(n_2)} \\ &= 0, \end{aligned} \tag{24}$$

where we take into account that the sum over n_2 in the expression above is zero for $k\neq l$. Therefore the negative value of $S_c^{(n_1n_2)}$ at $|E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha}|>\Gamma$ must be compensated by its positive value near the maximum. The sum rule (24) allows one to make a rough estimate of S_c near the maximum of S_d (and $\overline{M^2}$).

Let us assume that $S_c=R_mS_d$ at $|E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha}|<\Gamma/2$, whereas $S_c=R_tS_d$ at $|E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha}|>\Gamma/2$ [R_t is given by Eq. (22)]. The distribution of $S_d^{(n_1n_2)}$ can be reasonably approximated by the Breit-Wigner shape (see Secs. I and IV),

$$S_d^{(n_1n_2)}=\frac{A}{E^2+\Gamma^2/4}, \tag{25}$$

where $E=E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha}$, and $\Gamma=\Gamma_{n_1}+\Gamma_{n_2}$. The sum rule (24) implies that

$$R_m\int_0^{\Gamma/2}\frac{dE}{E^2+\Gamma^2/4}+R_t\int_{\Gamma/2}^{\infty}\frac{dE}{E^2+\Gamma^2/4}=0. \tag{26}$$

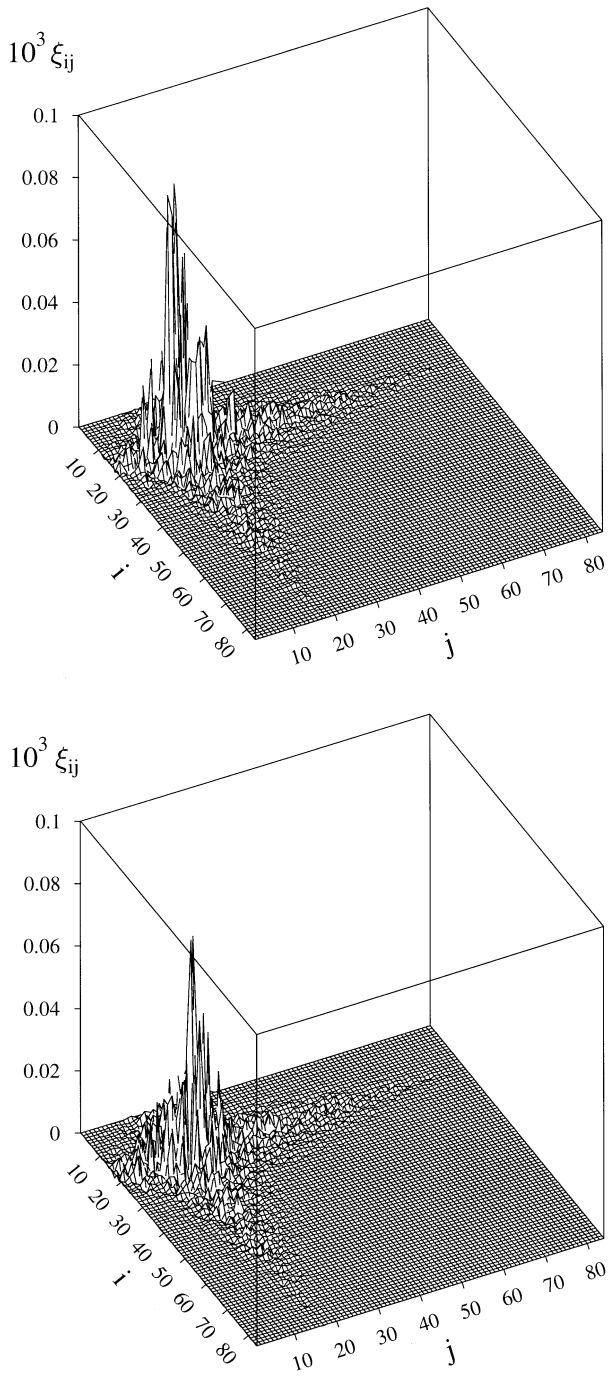

FIG. 3. The distribution of the items of the sum (15), $\xi_{ij}=C_i^{(n_1)}C_j^{(n_1)}C_{i'}^{(n_2)}C_{j'}^{(n_2)}$, for $n_1=55$, $n_2=66$, obtained in the TBRIM for the same set of parameters as in Fig. 1, averaged over $N_r=100$ realizations of V. Indices i and j in the figure run over those 84 components in which α is occupied and β is vacant. (a) Positive values. (b) Negative values (absolute values).

Since the two integrals in the above equation are equal, we have $R_m=-R_t$. Thus near the maximum the correlation contribution S_c is positive and enhances the squared matrix element with respect to the diagonal contribution,

$$\frac{S_d+S_c}{S_d}=1+R_m=2-(1+R_t)\simeq 2\left[1-\frac{m}{n(m-n)}\right]. \tag{27}$$

Comparing the values of the ratio S_c/S_d at the maximum and at the tail in Fig. 1(b) ($n=4,m=11$), one can see that indeed, $R_m\approx -R_t$. For larger n and m the correlation enhancement factor asymptotically reaches its maximal value of 2. The numerical example in Fig. 2 ($n=7,m=14$) shows the enhancement of $\overline{M^2}$ with respect to S_d at the maximum even greater in size than that predicted by Eq. (27). This is not too surprising since in Eqs. (25)–(27) we estimated the average value of R_m over an interval $\Delta E\simeq\Gamma$ around the maximum rather than the peak value at the maximum.

A similar estimate of S_c near maximum can be obtained by the direct calculation of the small component contribution to S_c (15). On an assumption that there are no correlations between principal components of compound states we can separate the contribution of small components. For example, in the resonance situation, $E^{(n_2)}-E^{(n_1)}\simeq\omega_{\beta\alpha}$, if the components $S_j^{(n_1)}$ and $S_{j'}^{(n_2)}$ are small ($|E_j-E^{(n_1)}|>\Gamma$, and consequently, $|E_{j'}-E^{(n_2)}|>\Gamma$), then they contain contributions proportional to the principal components $C_i^{(n_1)}$ and $C_{i'}^{(n_2)}$ [see Eqs. (16), (17)]. Analogously, $S_j^{(n_1)}$ and $S_{j'}^{(n_2)}$ may be among the principal components, and then the small components $C_i^{(n_1)}$ and $C_{i'}^{(n_2)}$ will contain correlated contributions. Thus we have the following estimate:

$$S_c^{(n_1n_2)} \simeq 2\widetilde{\sum_i}{}' \sum_{\text{small } j}{}' |C_i^{(n_1)}|^2|C_{i'}^{(n_2)}|^2\frac{\overline{H_{ij}H_{i'j'}}}{(E^{(n_1)}-E_j)(E^{(n_2)}-E_{j'})}. \tag{28}$$

Since $E_{j'}-E_j=E_{i'}-E_i\simeq E^{(n_2)}-E^{(n_1)}$ for the principal components i and i', $(E^{(n_1)}-E_j)$ and $(E^{(n_2)}-E_{j'})$ in the denominator always have the same sign, and S_c is positive (recall that $\overline{H_{ij}H_{i'j'}}>0$). Expression (28) can be estimated using the well known formula for the spreading width, $\Gamma=2\pi\overline{H_{ij}^2}/D$, where D is the mean level spacing for the many-body states. This yields $S_c\sim S_d$, in agreement with the previous estimate (27).

III. CORRELATIONS BETWEEN TRANSITION AMPLITUDES

We have shown that correlations between eigenvector components in a system with a two-body interaction between particles must be taken into account when calculating the variance of a matrix element between compound states $\overline{M^2}$. Another question is whether the above correlations between eigenstate components lead to correlations between different matrix elements,

$$M_{n_1n_2}M_{n_2n_3}=\langle n_1|\rho_{\alpha\beta}|n_2\rangle\langle n_2|\rho_{\beta\alpha}|n_3\rangle = \sum C_i^{(n_1)}C_j^{(n_2)}C_k^{(n_2)}C_l^{(n_3)}\langle i|\rho_{\alpha\beta}|j\rangle\langle k|\rho_{\beta\alpha}|l\rangle. \tag{29}$$

Our analysis shows that the correlations of the type (29) are absent, i.e., $\overline{M_{n_1n_2}M_{n_2n_3}}=\overline{M_{n_1n_2}}\;\overline{M_{n_2n_3}}=0$. The result could

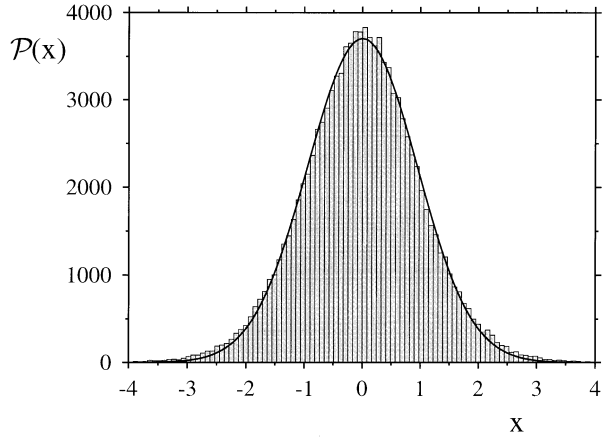

FIG. 4. Probability density of the normalized matrix elements $x \equiv \langle n_1|\rho_{\alpha\beta}|n_2\rangle / (\overline{|\langle n_1|\rho_{\alpha\beta}|n_2\rangle|^2})^{1/2}$ in the TBRIM for the parameters of Fig. 1. The histogram is obtained for $N_r = 5$ Hamiltonian matrices. Solid curve is the normalized Gaussian distribution.

be different if the principal components of different eigenvectors ($|n_1\rangle$ and $|n_3\rangle$) were correlated. This effect takes place in the separable interaction model [5,4], but we have not found such correlations in the TBRIM.

The absence of correlations between different amplitudes is confirmed by direct numerical experiments. First, we have studied the probability density of the matrix elements $M_{n_1 n_2}$ for different n_1, n_2 obtained for a number of realizations of the two-body matrix elements $V_{\alpha\beta\gamma\delta}$. Since the variance of $M_{n_1 n_2}$ depends on n_1 and n_2, the probability density of $M_{n_1 n_2}$ has been obtained by normalizing each matrix element $M_{n_1 n_2}$ to its root-mean-squared value which was calculated by averaging over the realizations of $V_{\alpha\beta\gamma\delta}$. The resulting probability density $\mathscr{P}(M_{n_1 n_2} / \sqrt{\overline{M^2_{n_1 n_2}}})$, averaged over $N_r = 5$ realizations of $V_{\alpha\beta\gamma\delta}$, turns out to be quite close to Gaussian (Fig. 4). This result follows from the fact that each matrix element between compound states is the sum of a large number of random (or almost random) terms, see Eq. (10), so that the central limit theorem applies. Therefore the correlations found in the preceding section do not show up, unless more complicated correlations involving *different* components of the *same* eigenstate, like those in Eqs. (8) or (11), are probed.

To check whether some correlations between different matrix elements (29) exist, we have plotted the matrix element $\langle n_1|\rho_{\alpha\beta}|n_2\rangle$ versus another one, $\langle n_1|\rho_{\alpha\beta}|n_2 - 1\rangle$, where $|n_2 - 1\rangle$ is the eigenstate immediately preceding $|n_2\rangle$, for some fixed n_1, n_2, α, and β, obtained from $N_r = 387$ different Hamiltonian matrices [Fig. 5(a)]. The latter were generated by using different random realizations of $V_{\alpha\beta\gamma\delta}$. Detailed analysis of the distribution of the points in this figure does not reveal any sort of correlations.

The next question is the existence of correlations between matrix elements $M_{n_1 n_2}$ and $W_{n_1 n_2}$ of different operators for the same compound states $|n_1\rangle$ and $|n_2\rangle$. If the expansions of these matrix elements [see Eq. (10)] contain identical matrix elements of the density matrix operator $\rho_{\alpha\beta}$, such correlations, in principle, do exist:

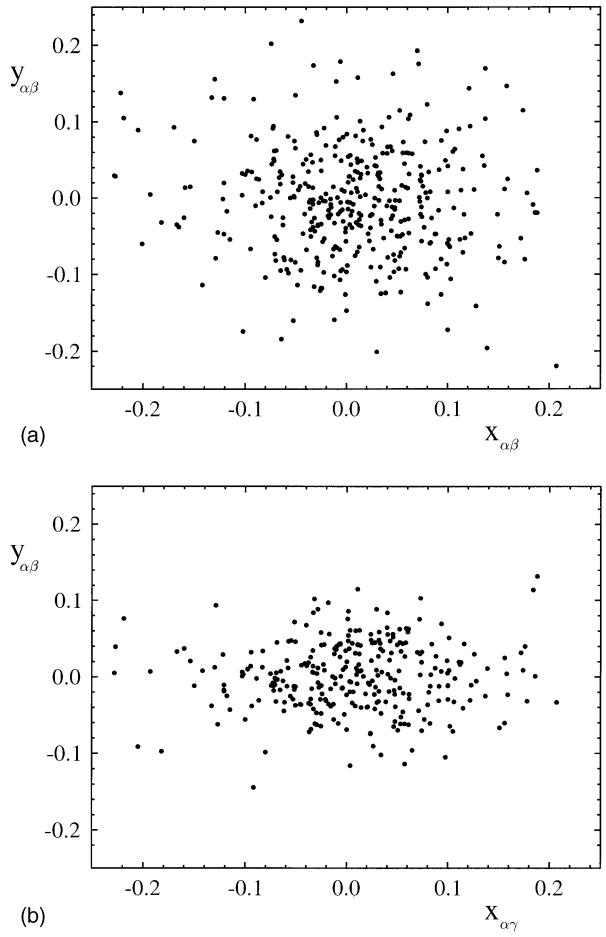

FIG. 5. (a) The matrix element $y_{\alpha\beta} = \langle n_1|\rho_{\alpha\beta}|n_2\rangle$ for $n_1 = 55$, $n_2 = 67$, plotted vs the matrix element $x_{\alpha\beta} = \langle n_1|\rho_{\alpha\beta}|n_3\rangle$ for $n_3 = 66$; $\alpha = 4$, $\beta = 5$, and other TBRIM parameters as in Fig. 1. The number of points in the figure is $N_r = 387$. No evidence of correlations between x and y is present. (b) The matrix element $y_{\alpha\beta} = \langle n_1|\rho_{\alpha\beta}|n_2\rangle$ for $\alpha = 4$, $\beta = 6$ vs $x_{\alpha\gamma} = \langle n_1|\rho_{\alpha\gamma}|n_2\rangle$ with $\gamma = 5$ for $n_1 = 55$ and $n_2 = 66$. Again, there is no indication of correlations between x and y. Note the difference in the vertical and horizontal scale due to the fact that for given n_1 and n_2 the energy difference $E^{(n_1)} - E^{(n_2)}$ is approximately in resonance for the transition between $\alpha = 4$ and $\beta = 5$ and off resonance for $\alpha = 4$ and $\gamma = 6$.

$$\overline{M_{n_1 n_2} W_{n_2 n_1}} = \sum_{\alpha,\beta} M_{\alpha\beta} W_{\beta\alpha} \overline{\langle n_1|\rho_{\alpha\beta}|n_2\rangle^2} \neq 0. \qquad (30)$$

A more complicated question is whether the matrix elements of different ''elementary'' transition operators $\rho_{\alpha\beta}$ and $\rho_{\gamma\delta}$ are indeed uncorrelated [as we assumed writing Eq. (30)]. The product of such two matrix elements can be presented in the form

$$\langle n_1|\rho_{\alpha\beta}|n_2\rangle\langle n_2|\rho_{\delta\gamma}|n_1\rangle = \sum_{i,j,k,l} C_i^{(n_1)} C_j^{(n_1)} C_k^{(n_2)} C_l^{(n_2)} \times \langle i|\rho_{\alpha\beta}|k\rangle\langle l|\rho_{\delta\gamma}|j\rangle \qquad (31)$$

$$\simeq 2 \sum_{i,j'}{}' \frac{|C_i^{(n_1)}|^2 |C_{j''}^{(n_2)}|^2 H_{ij} H_{i'j''}}{(E^{(n_1)} - E_j)(E^{(n_2)} - E_{i'})}, \qquad (32)$$

where the last expression is written for n_1 and n_2 far from the maximum ($|E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha}|>\Gamma, |E^{(n_2)}-E^{(n_1)}-\omega_{\delta\gamma}|>\Gamma$), and $|i'\rangle=a^\dagger_\beta a_\alpha|i\rangle$, $|j''\rangle=a^\dagger_\delta a_\gamma|j\rangle$. It can be shown that in our model $\overline{H_{ij}H_{i'j''}}\propto\delta_{\alpha\gamma}\delta_{\beta\delta}$. Therefore there are no terms in the expression (32) which would give nonzero contributions, and the average of (31) is zero. The absence of correlations in this case is illustrated by Fig. 5(b), where numerical data obtained in the TBRIM are presented. As in the case of the matrix elements between different pairs of compound states, no correlations can be seen between the matrix elements of different transition operators.

IV. STATISTICAL DESCRIPTION OF THE TRANSITION AMPLITUDES

In this section we use the TBRIM to test the validity of the statistical approach to the calculation of transition amplitudes between compound states of complex systems developed in [3,12,20]. In what follows we first outline the main ideas of the statistical approach. The variance of the matrix elements of an operator $\hat{M}$ (9) between the compound states $|n_1\rangle$ and $|n_2\rangle$ can be presented in the following form [compare with Eq. (30)]:

$$\overline{|M_{n_1n_2}|^2}=\sum_{\alpha,\beta}|M_{\alpha\beta}|^2\overline{|\langle n_1|\rho_{\alpha\beta}|n_2\rangle|^2}, \tag{33}$$

where we have taken into account the result of the preceding section that the average of the correlator (31) is zero unless $\gamma=\alpha,\delta=\beta$. Therefore the calculation of $\overline{|M_{n_1n_2}|^2}$ (or $\overline{M_{n_1n_2}W_{n_2n_1}}$) is reduced to the calculation of $\overline{|\langle n_1|\rho_{\alpha\beta}|n_2\rangle|^2}$.

It was suggested in Sec. II that $\overline{|\langle n_1|\rho_{\alpha\beta}|n_2\rangle|^2}$ can be presented as the sum of the diagonal and correlational sums S_d and S_c, Eqs. (11)–(13). Since we have already estimated the ratio $(S_d+S_c)/S_d$, it is enough to calculate only S_d, Eq. (12). Following [12] let us replace the squared components $|C_i^{(n_1)}|^2$ and $|C_k^{(n_2)}|^2$ by their average values,

$$\overline{|C_i^{(n_1)}|^2}\equiv w(E_i,E^{(n_1)}),\quad \overline{|C_k^{(n_2)}|^2}\equiv w(E_k,E^{(n_2)}), \tag{34}$$

where the averaging goes as usual either over a number of realizations of the two-body interaction matrix elements (ensemble average), or over a number of neighboring eigenstates (physical energy average); in the spirit of ergodicity the results are presumably the same. The function w is proportional to the strength function introduced by Wigner [10], which is also called the local spectral density of states. Note that definition (34) also implies that the mean-square contribution of the component i in the eigenstate n_1 is determined by their energies, E_i and $E^{(n_1)}$ (in fact, by their difference $|E_i-E^{(n_1)}|$). For states localized in the given basis, w is a bell-shaped function with a typical width determined by the spreading width Γ. There is some theoretical and experimental evidence that it can be approximated by the Breit-Wigner formula, although its tails decrease faster than $|E_i-E^{(n_1)}|^{-2}$ (see references in Sec. I).

The diagonal sum now takes the form

$$S_d^{(n_1n_2)}=\sum_{i,k}w(E_i,E^{(n_1)})w(E_k,E^{(n_2)})\langle i|\rho_{\alpha\beta}|k\rangle\langle k|\rho_{\beta\alpha}|i\rangle. \tag{35}$$

The summation over k for a fixed i includes only one state, $|k\rangle=\rho_{\beta\alpha}|i\rangle$, with $E_k=E_i+\omega_{\beta\alpha}$. On the other hand, we can write

$$\sum_k\langle i|\rho_{\alpha\beta}|k\rangle\langle k|\rho_{\beta\alpha}|i\rangle=\langle i|\rho_{\alpha\beta}\rho_{\beta\alpha}|i\rangle=\langle i|\hat{n}_\alpha(1-\hat{n}_\beta)|i\rangle, \tag{36}$$

where $\hat{n}_\alpha=a^\dagger_\alpha a_\alpha$ and $\hat{n}_\beta=a^\dagger_\beta a_\beta$ are the occupation number operators. Thus we obtain

$$S_d^{(n_1n_2)}=\sum_i w(E_i,E^{(n_1)})w(E_i+\omega_{\beta\alpha},E^{(n_2)})\times\langle i|\hat{n}_\alpha(1-\hat{n}_\beta)|i\rangle. \tag{37}$$

The matrix element $\langle i|\hat{n}_\alpha(1-\hat{n}_\beta)|i\rangle$ is equal to 1 if the orbital α is occupied and β is vacant in the basis state $|i\rangle$, otherwise, it is zero. We used this fact earlier [Eq. (14)] to reduce the summation to these states only. Now we proceed in a different way. Both w's in (37) are smooth functions of energy normalized as $\Sigma_i w(E_i,E^{(n_1)})=1$. This allows one to replace the matrix element of $\hat{n}_\alpha(1-\hat{n}_\beta)$ by its expectation value,

$$\begin{aligned}\overline{\langle i|\hat{n}_\alpha(1-\hat{n}_\beta)|i\rangle}&=\sum_i w(E_i,E^{(n_1)})\langle i|\hat{n}_\alpha(1-\hat{n}_\beta)|i\rangle\\&=\sum_i\overline{|C_i^{(n_1)}|^2}\langle i|\hat{n}_\alpha(1-\hat{n}_\beta)|i\rangle\\&\simeq\langle\hat{n}_\alpha(1-\hat{n}_\beta)\rangle_{n_1}.\end{aligned} \tag{38}$$

The sign $\simeq$ above is a reminder that the left-hand side is the local average over the states $|n_1\rangle$. Practically, when the number of components is large, the fluctuations of $\langle\hat{n}_\alpha(1-\hat{n}_\beta)\rangle_{n_1}$ are expected to be small. Now we can rewrite Eq. (37) in a form similar to Eq. (14), but without any restrictions on the summation variable i,

$$S_d^{(n_1n_2)}=\langle\hat{n}_\alpha(1-\hat{n}_\beta)\rangle_{n_1}\sum_i w(E_i,E^{(n_1)})w(E_i+\omega_{\beta\alpha},E^{(n_2)}). \tag{39}$$

It was shown in [12] that under some reasonable assumptions about the functions w one can introduce a ''spread δ function'' $\tilde{\delta}(\Delta)$,

$$\begin{aligned}\tilde{\delta}(\Delta)&=D_2^{-1}\sum_i w(E_i,E^{(n_1)})w(E_i+\omega_{\beta\alpha},E^{(n_2)})\\&=D_2^{-1}\int\frac{dE_i}{D_1}w(E_i,E^{(n_1)})w(E_i+\omega_{\beta\alpha},E^{(n_2)}),\end{aligned} \tag{40}$$

where $\Delta=E^{(n_2)}-E^{(n_1)}-\omega_{\beta\alpha}$, and D_1 and D_2 are local mean level spacings for the n_1 and n_2 eigenstates. The function $\tilde{\delta}(\Delta)$ is symmetric, its characteristic width is determined by the spreading widths of the eigenstates n_1 and n_2,

$\Gamma \sim \Gamma_1 + \Gamma_2$, and it is normalized to unity, $\int \widetilde{\delta}(\Delta) d\Delta = 1$, just as the standard δ function. If w's have Breit-Wigner shapes, $\widetilde{\delta}$ is also a Breit-Wigner function with $\Gamma = \Gamma_1 + \Gamma_2$. The fact that $S_d^{(n_1 n_2)}$ is proportional to the function $\widetilde{\delta}(\Delta)$ is a particular manifestation of the energy conservation for transitions between the quasistationary basis states [20] [if $\Gamma \to 0$, then $\widetilde{\delta}(\Delta) \to \delta(\Delta)$]. Using Eqs. (33), (39), and (40) we can finally present the diagonal contribution to the variance of the matrix element $M_{n_1 n_2}$ in the form

$$\overline{|M_{n_1 n_2}|^2_{\text{diag}}} = \sum_{\alpha,\beta} |M_{\alpha\beta}|^2 \langle \hat{n}_\alpha (1 - \hat{n}_\beta) \rangle_{n_1} \times D_2 \widetilde{\delta}(E^{(n_2)} - E^{(n_1)} - \omega_{\beta\alpha}). \quad (41)$$

This expression is apparently asymmetric with respect to the states n_1 and n_2. By performing the calculation in a different way we can obtain

$$S_d^{(n_1 n_2)} = \langle \hat{n}_\beta (1 - \hat{n}_\alpha) \rangle_{n_2} \sum_k w(E_k - \omega_{\beta\alpha}, E^{(n_1)}) w(E_k, E^{(n_2)}), \quad (42)$$

instead of Eq. (39), and thereby arrive at a different formula for the variance,

$$\overline{|M_{n_1 n_2}|^2_{\text{diag}}} = \sum_{\alpha,\beta} |M_{\alpha\beta}|^2 \langle \hat{n}_\beta (1 - \hat{n}_\alpha) \rangle_{n_2} \times D_1 \widetilde{\delta}(E^{(n_2)} - E^{(n_1)} - \omega_{\beta\alpha}), \quad (43)$$

where the occupancies factor is now calculated for the state n_2 (it represents the probability to find the orbital β occupied, and α empty). If the suppositions made in the above derivations are correct, the two formulas (41) and (43) should give identical results.

In the present work we use the TBRIM to check the accuracy of the statistical approach described above. Figure 6(a) presents a comparison between the values of $S_d^{(n_1 n_2)}$ as given by Eqs. (39), (42), and those from the initial expression (12). Clearly, there is a good agreement between the three formulas.

It is quite important for applications of the statistical approach (see [4,20]) that further simplifications be made by replacing the correlated occupancies product $\langle \hat{n}_\alpha \hat{n}_\beta \rangle_{n_1}$ in Eq. (38) by the product of the two mean values, $\langle \hat{n}_\alpha \rangle_{n_1} \langle \hat{n}_\beta \rangle_{n_1}$. This is definitely a valid operation when the numbers of excited particles and active orbitals are large, so that the occupation numbers for different orbitals become statistically independent. Then one would be able to use the relation

$$\langle \hat{n}_\alpha (1 - \hat{n}_\beta) \rangle \simeq n(\epsilon_\alpha)[1 - n(\epsilon_\beta)], \quad (44)$$

where $n(\epsilon_\alpha)$ and $n(\epsilon_\alpha)$ are the occupation numbers. (They can be calculated, e.g., using the Fermi-Dirac formula with an effective temperature, see [17,20]; see also [9] where the relation between thermalization and chaos is studied in nuclear shell-model calculations.) The result of such simplification is shown in Fig. 6(b), where the diagonal contribution (12) is again compared with the values obtained from Eqs. (39), (42), using approximation (44). In spite of the fact that the TBRIM calculation included $n = 4$ particles only, the agreement remains quite reasonable, the error being about 10%. To examine the quality of the approximation at the tails of the distribution, Fig. 7 shows the ratio of S_d as given by Eqs. (39), (42) to the directly calculated diagonal term S_d, Eq. (12). The difference between Figs. 7(a) and 7(b) highlights the inaccuracy introduced by an additional approximation (44) for the occupation numbers.

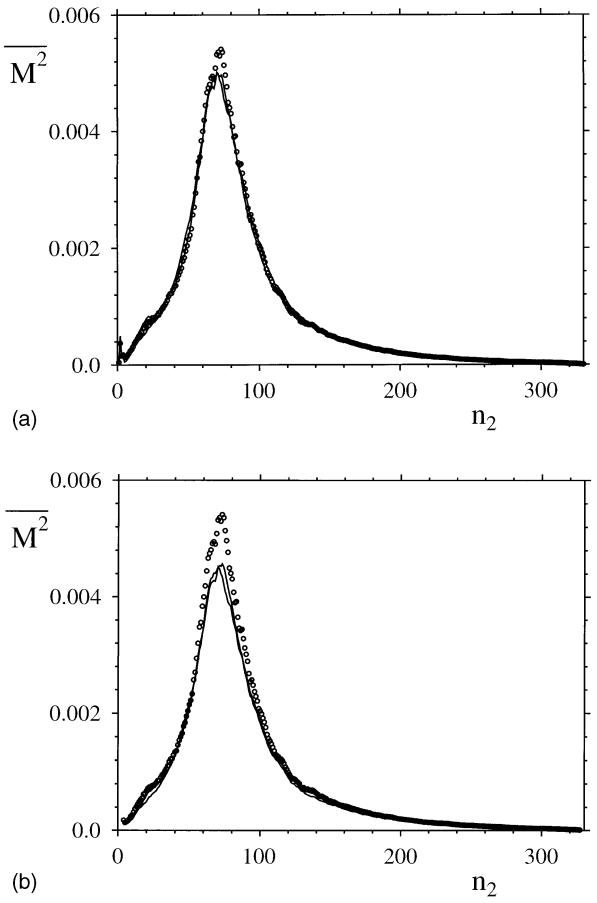

FIG. 6. (a) The diagonal contribution to the mean-square matrix element as obtained from Eqs. (39), (42) (solid lines) in comparison with the direct calculation of S_d, Eq. (12) (circles). The TBRIM parameters are the same as in Fig. 1. (b) Same as (a), with the occupancy factors in Eqs. (39), (42) calculated by means of Eq. (44).

In order to make a more direct test of the validity of substitution (44), we plotted in Fig. 8 the correlator $\langle \hat{n}_\alpha \hat{n}_\beta \rangle_{n_1} / [\langle \hat{n}_\alpha \rangle_{n_1} \langle \hat{n}_\beta \rangle_{n_1}]$ as a function of n_1. Consistent with the small number of particles, this correlator displays large fluctuations; however, its average value of about 0.8 is still rather close to 1.

V. SPREADING WIDTHS FOR DIFFERENT BASIS COMPONENTS

In Sec. IV when considering the statistical approach to the calculation of the variance of matrix elements between compound states, it was assumed that the spreading widths Γ are the same for all basis components. However, this question is not trivial. As is discussed in the literature, the spreading

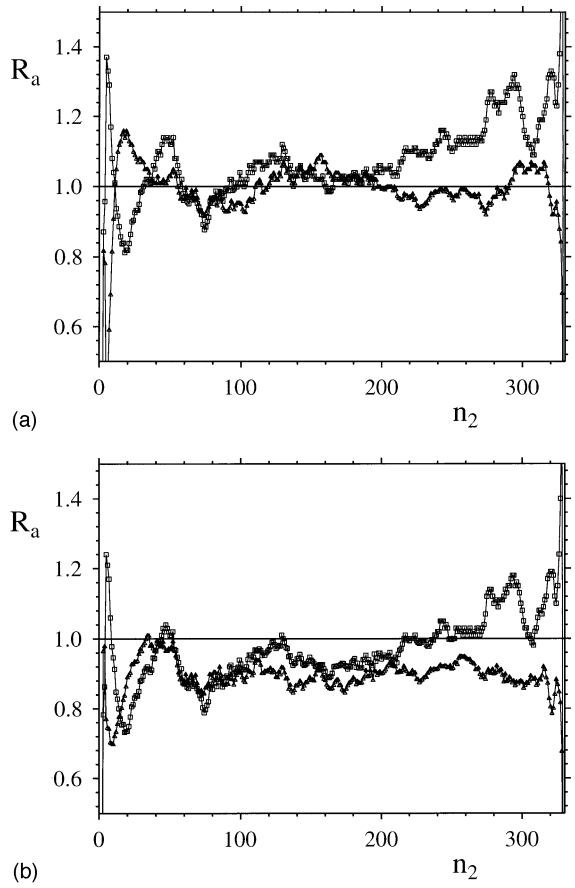

FIG. 7. (a) The ratio R_a of the approximation represented in Fig. 6(a) by the solid lines, to the value of S_d. (b) Same as in Fig. 7(b) for the data of Fig. 6(b).

widths of components corresponding to different numbers of excited particles could have significantly different values. For example, in [22] it is argued that two-particle–one-hole states ($2p$-$1h$) can lead to correlations between values of parity nonconserving effects [1], if the spreading width of

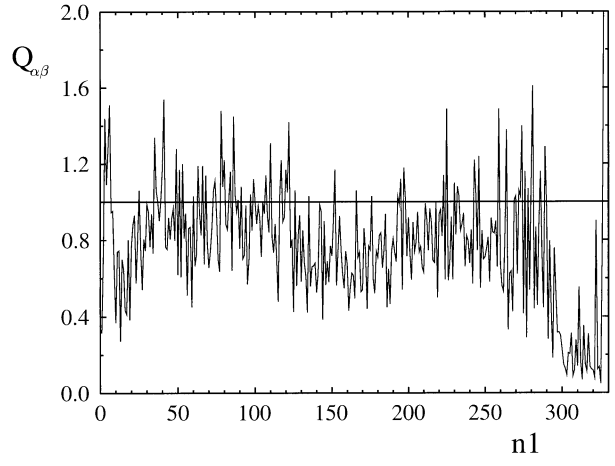

FIG. 8. The correlator $Q_{\alpha\beta}\equiv\langle\hat{n}_\alpha\hat{n}_\beta\rangle_{n_1}/[\langle\hat{n}_\alpha\rangle_{n_1}\langle\hat{n}_\beta\rangle_{n_1}]$ vs the eigenstate number n_1. The TBRIM parameters are the same as in Fig. 1. The average value of the correlator is about 0.8, which means that the correlations between the occupancies of different orbitals are not very strong.

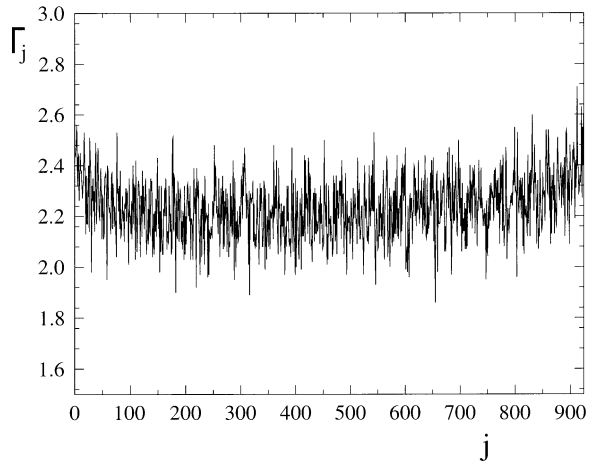

FIG. 9. The spreading width Γ_j calculated as the rms deviation from the center of the distribution of the components $|C_j^{(n_1)}|^2$ for each basis state j. The data are obtained for one matrix H_{ij} corresponding to $n=6$ particles and $m=12$ orbitals.

the $2p$-$1h$ states is two orders of magnitude smaller than that of $1p$ states. In such a case one might expect that in our model the spreading width would show a rapid decrease as a function of the number of excited particles in the basis component.

To study this question in detail, we have performed additional tests. In Fig. 9 the root-mean-squared spreading width Γ_j for all basis states $|j\rangle$ is presented for $n=6$, $m=12$ ($N=924$). Here we use the following definition:

$$\Gamma_j^2=\langle j|(H-\langle j|H|j\rangle)^2|j\rangle, \tag{45}$$

which can also be presented in several equivalent forms,

$$\Gamma_j^2=(H^2)_{jj}-(H_{jj})^2 \tag{46}$$

$$=\sum_{i\neq j}H_{ij}^2 \tag{47}$$

$$=M_2-(M_1)^2, \tag{48}$$

the last one relating Γ_j directly to the moments M_p of the strength function $\rho_w(E,j)\equiv\Sigma_n|C_j^{(n)}|^2\delta(E-E^{(n)})$,

$$M_p=\int\rho_w(E,j)E^p dE=\sum_n|C_j^{(n)}|^2(E^{(n)})^p. \tag{49}$$

Equations (46)–(48) can be obtained using closure, $\Sigma_j|j\rangle\langle j|=\Sigma_n|n\rangle\langle n|=1$, where $|j\rangle$ and $|n\rangle$ are the basis state and the eigenstate, respectively, $\langle j|n\rangle\equiv C_j^{(n)}$. We should note that the rms spreading width is different from that introduced intuitively in Sec. IV as the characteristic width of the strength function. For example, if the strength function has a Breit-Wigner form, its second and higher moments are infinite. In Wigner's BRM the rms spreading width is determined by the bandwidth b as $\Gamma_j=\sqrt{2bV^2}$, whereas the ''Breit-Wigner spreading width'' is $\Gamma_{\rm BW}=2\pi v^2/D$ (see Introduction). However, in a more realistic situation the strength function drops rapidly, its second moment is finite, and the difference between the rms Γ and $\Gamma_{\rm BW}$ is not large.

From Fig. 9 one can see that apart from small natural fluctuations, the rms spreading width is the same for all components. To exclude a weak dependence of Γ_j on the energy of the basis state j (boundary effects seen as rises of Γ_j at small and large j), we calculate the mean value $\overline{\Gamma}$ and the rms deviation $\delta\Gamma$ of the spreading width by averaging over $j=50$–874. The results are $\overline{\Gamma}\approx 2.22$, $\delta\Gamma\approx 0.12$. The latter value shows that the fluctuations of the width are very small. This result is in agreement with computations made for the Ce atom [12]; similar results have been recently obtained in the nuclear sd-shell-model calculations [9,23]. The fluctuations of the width are small due to the large number of ''decay channels'' for each basis component (each component is coupled to many others by random interaction) [4]. Formally, this can be obtained from Eq. (47). For example, in the TBRIM one obtains

$$\overline{\Gamma^2}=(n-1)V^2K_1+V^2K_2=K_{12}V^2, \tag{50}$$

which shows that for a large number of ''independent decay channels,'' $K_{12}=(n-1)K_1+K_2$ [see Eq. (3)], the statistics of Γ_j is given by the χ^2 distribution with $K_{12}\gg 1$ degrees of freedom, resulting in the $\sim 1/\sqrt{K_{12}}$ decrease of fluctuations. More accurately, the relative rms fluctuation of the squared width (45) is given by $\delta(\Gamma^2)/\sqrt{\overline{\Gamma^2}}=\sqrt{2/K_{12}}$. For $n=6$, $m=12$, Eq. (50) yields $\sqrt{\overline{\Gamma^2}}\approx 2.41$, and the relative fluctuation is 0.07. These values are close to the numerical ones quoted above (the discrepancy is mainly due to the difference between the mean width and its rms value, and the corresponding difference in the fluctuations of Γ_j and Γ_j^2).

To check the independence of Γ_j of the number of excited particles in the component j, we have calculated the average spreading width $\overline{\Gamma(p)}$ for basis states with a fixed number of excited particles p, $p=1,2,\ldots,n-1$. In the numerical experiment shown in Fig. 6 all $\overline{\Gamma(p)}$ for $p=1$–5 proved to be approximately the same, $\overline{\Gamma(p)}\approx 2.2$.

The above consideration shows that the statistical approach does not provide any support for the dependence of the spreading width on the number of excited particles. This indicates that the argumentation in favor of a strong dependence, based on different decay phase volumes for different numbers of excited particles, seems to be incorrect. In our opinion, the difference in the spreading widths could appear as a result of specific dynamical effects. For example, this could be an influence of levels in other potential wells which appear at higher nuclear deformation, or due to an interaction with collective motions, such as rotations and vibrations.

VI. CONCLUSIONS

The calculation of the mean-square matrix element of an operator between compound states of a many-body system has been considered. We have shown that the two-body nature of the interaction between particles manifests itself in the existence of correlations between the components of the ''chaotic'' compound eigenstates. These correlations taken together with the correlations between the many-particle Hamiltonian matrix elements result in a relatively large correlation contribution to the mean-square matrix element. The correlations exist even if the two-body matrix elements are independent random variables, as in the TBRIM. Such correlations can be understood in terms of the perturbative mixing of the distant small components to the principal components of the eigenstates. If the Hamiltonian matrix elements are random variables the correlations of this type vanish.

One of the most interesting features of the correlations found in our work is that they do not decrease with the increase of the number of excited particles or active orbitals. Thus they must be taken into account when calculating matrix elements of a weak interaction between compound states in nuclei.

Another feature concerns the shape of the distribution of the density matrix operator near its maximum. As one can see from Figs. 1(a) and 2(a), the correlations create a sharp spikelike form of the distribution, instead of a smooth Gaussian or Breit-Wigner form. With such sharp peaks, the strength function for any particular operator $\hat{M}$ can have the so-called gross structure, due to many single-particle transition terms in the expression (33). Without these specific correlations, the strength function would be much smoother and the gross structure would not be seen. It is also interesting to note that there are very large mesoscopic-type fluctuations in the distribution near the maximum, depending on a specific (random) realization of the two-body interaction V. This fact is also the consequence of strong correlations.

Our study also demonstrated that the spreading widths of different basis components are approximately constant and fluctuate very weakly. In particular, we have not found any dependence on the number of excited particles in the component.

The statistical approach to the calculations of such matrix elements has been tested in the present work with the help of the TBRIM. The numerical results obtained in this work support the validity of the statistical approach. The TBRIM has also enabled us to check that the matrix elements of different transition operators between a pair of compound states are uncorrelated, as are the matrix elements of a given operator between different pairs of compound states.

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