Probability and Distribution Theory
SOR201
Introduction

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I am taking over from Dr Donald Davison, and will make use of his typed set of lecture notes, problem sheets and past exam papers.

Lectures, rm 1022

Mon	3–4 pm	
Tue	3–4 pm	
Fri	3–4 pm	$\longrightarrow$ Thu 1–2 pm (?)

Tutorials (Examples classes)

Mon 2–3 pm, rm 3001

Homework problem sheets

Handed out on Thursday, handed in Thursday week

"Open class" (*Your* questions about lectures, etc.)

Friday 3–4 pm?

Web site `http://www.am.qub.ac.uk/users/g.gribakin/Teaching.html`

Under construction

rm 4012, David Bates building *(Welcome!)*

0 Introduction: Combinatorial Analysis

0.1 Problems

1. Car number plates consist of three letters and four digits, and looks like XXX YYYY, where X are letters, and Y are digits.

   (a) How many different number plates can one have?

   (b) What is the answer if repetitions among letters or digits are prohibited?

2. Twelve performers are to appear one by one in the final of a singing contest.

   In how many ways can this be arranged?

3. At Level 3 the departments of Pure and Applied Mathematics offer 3 and 4 modules, respectively, in each semester.

   (a) In how many ways can a student choose his/her three first semester modules?

   (b) What is the total number of different enrolment schemes for the whole year?

0.2 Some more problems

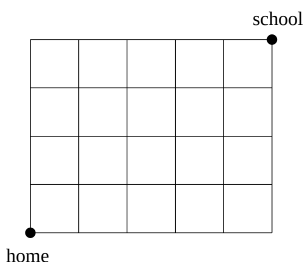
1. Prove formally the identity
$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}.$$

2. Prove that
$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \dots \binom{n}{r}\binom{m}{0}.$$
[Hint: consider a group of n men and m women. How many different groups of size r are possible?]

3. Show that
$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0.$$
Hint: use the binomial theorem.

4. Consider a group of n people. If everyone shakes hands with everyone else, how many shakehands take place?

   This problem is close (but not identical) to another one: how many diagonals are in a regular n-sided polygon? (A diagonal connects any two vertices which do not have a side in common.)

5. 

The diagram is the street map of a town. The side of each square on the map is 100 m, and it is clear that to get from home to school a student needs to walk 900 m. How many different paths of this length can he choose? [Hint: between the crossroads the student can move only to the right or upwards on the diagram.]

6. How many different letter arrangements can be made from the words (a) MATHS, (b) STRESS, and (c) MISSISSIPPI?

7. Someone wants to invest £20,000 in four different investments. Each investment must be in units of £1,000. How many different investment strategies are possible?

8. Twelve thieves sit around a campfire. Their chief needs to choose a team of 5 to go and hide the booty. In how many ways can he do this, if the thieves who sit next to each other are feuding (and hence cannot be members of the same team).

9. A flee jumps between adjacent integer points on the number line (e.g., from 0 it can jump to -1 or $+1$). Suppose that the flee starts from the origin and makes n jumps. In how many ways can it reach an integer point x? Of course, the answer is zero for $|x| > n$... [Hint: think of the numbers of jumps to the left and to the right, and make sure the flee ends up at x.]